Principle of Blackout Communications

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Introduction

THE present discussion is intended to outline the method of approach to the communications blackout problem in both conical and cylindrical geometries. The cause of the blackout results from the formation of a plasma sheath which surrounds the vehicle traveling at hypersonic speed. It is known in connection with radio-wave propagation through the ionosphere that, in dense ionized media (which is characterized by the plasma-electron frequency ω_p), a comparatively low frequency (i.e., $\omega < \omega_p$) signal cannot propagate. However, the radio-wave propagation again becomes possible in the presence of a static magnetic field in the plasma. Therefore, it is very natural for one to be inclined to the idea of somehow introducing a uniform magnetic field in the plasma medium. Actually, the idea of using a static magnetic field is not new, and there is a good deal of discussion on this subject in literature.[1] All of these discussions, however, pertain to simple geometries[2] (e.g., rectangular), which do not really resemble the actual shape of the space vehicle. Moreover, a satisfactory elimination of the blackout also depends considerably upon the orientation of the static magnetic field with respect to the antenna configuration. In order to understand the major causes of the blackout phenomena, we shall restrict ourselves only to simplified models of the plasma and the re-entry vehicle.

Conical Geometry

Here we shall consider a conical shape for the space vehicle with an applied static magnetic field in the azimuthal ϕ direction. The spherical system of coordinates is used to represent the cone, the axis of which is along the polar z-axis (see Fig. 1a). For the moment, we shall assume that such a uniform static magnetic field can in some way be implemented.

Using a linear theory of approximation, the dielectric constant of a stationary, cold plasma in a uniform static magnetic field in the angular ϕ direction can be represented by the following mathematical form:

$$\bar{\boldsymbol{\varepsilon}} = \mathbf{r}_0\mathbf{r}_0\epsilon_1 + \mathbf{r}_0\boldsymbol{\theta}_0 j\epsilon_2 - \boldsymbol{\theta}_0\mathbf{r}_0 j\epsilon_2 + \boldsymbol{\theta}_0\boldsymbol{\theta}_0\epsilon_1 + \boldsymbol{\phi}_0\boldsymbol{\phi}_0\epsilon_3 \quad (1)$$

where $\mathbf{r}_0$, $\boldsymbol{\theta}_0$, and $\boldsymbol{\phi}_0$ are unit vectors in spherical coordinates.

The preceding representation of $\bar{\boldsymbol{\varepsilon}}$ in (1) has been derived from the following relations:

$$\nabla \times \mathbf{H} = j\omega\epsilon_0\bar{\boldsymbol{\varepsilon}}\cdot\mathbf{E} = j\omega\epsilon_0\,\mathbf{E} - n_e q_e \mathbf{V}$$

$$m_e\,(\nu\mathbf{V} + j\omega\mathbf{V}) = -q_e\mathbf{E} + q_e B_0 \boldsymbol{\phi}_0 \times \mathbf{V}$$

where $\mathbf{V}$ is the a.c. velocity of the stationary electron-plasma. Here the components of the dielectric tensor are defined as

$$\epsilon_1 = 1 + \omega_p^2(1 - j\nu/\omega)/[\omega_c^2 - \omega^2(1 - j\nu/\omega)^2] \quad (2a)$$

$$\epsilon_2 = (\omega_c/\omega)\omega_p^2/[\omega_c^2 - \omega^2(1 - j\nu/\omega)^2] \quad (2b)$$

$$\epsilon_3 = 1 - \omega_p^2/[\omega^2(1 - j\nu/\omega)] \quad (2c)$$

where ν is the collision frequency in radians, $\omega_c = q_e B_0/m_e$ is the cyclotron frequency in radians, $\omega_p = [q_e^2 n_e/(\epsilon_0 m_e)]^{1/2}$ is the electron plasma frequency in radians, q_e is the magnitude of the charge of an electron, B_0 is the applied static magnetic-field induction, n_e is the electron density, $\mathcal{E}_0$ is the dielectric constant of vacuum, and m_e is the mass of an electron. In the derivation of the equivalent dielectric constant of the plasma, the motion of an ion due to an electromagnetic signal (with

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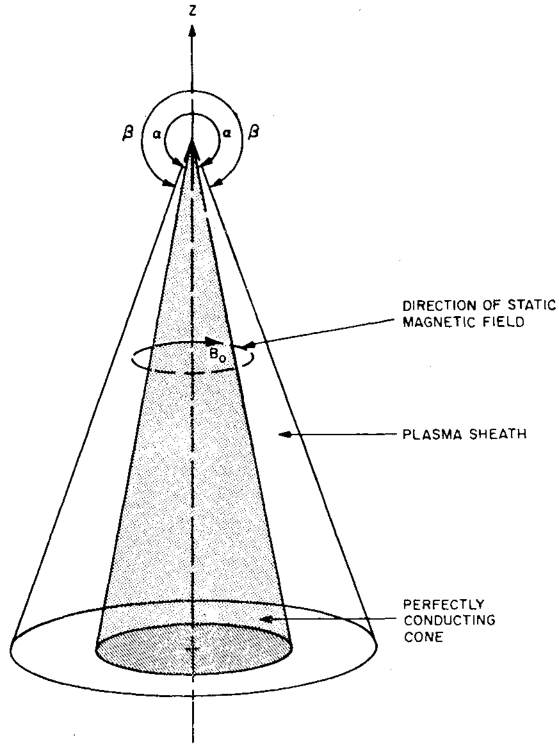

Fig. 1a Geometry of the conical structure.

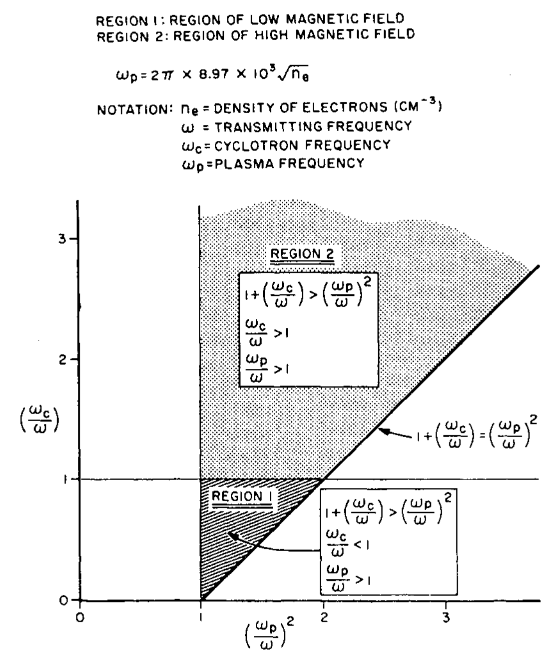

Fig. 1b Allowable frequency region for satisfactory communication during blackout in the presence of an applied static magnetic field.

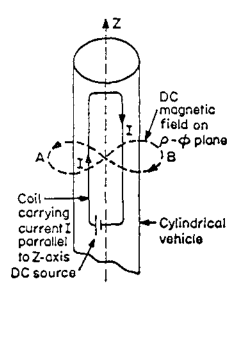

Fig. 1c Principle of generation of a d.c. magnetic field in the angular ϕ direction.

time-dependence $e^{j\omega t}$) is neglected in comparison to that of an electron. All the components of $\bar{\epsilon}$ are assumed to be independent of the coordinates (at least independent of the angle ϕ). For convenience, it will also be assumed that the source of the electromagnetic fields is constant in the ϕ direction such that $\partial/\partial\phi = 0$, and only the components E_r, E_θ, and H_ϕ are excited. This situation can be achieved by introducing either a magnetic ring source (slot antenna) in the azimuthal direction or an electric dipole along the z axis. With these considerations, Maxwell's equations in a source-free, anisotropic plasma region can be expressed in the following way:

$$\left.\begin{aligned} \frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) - \frac{1}{r}\frac{\partial}{\partial\theta}E_r &= -j\omega\mu_0 H_\phi \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, H_\phi) &= j\omega\epsilon_0(\epsilon_1 E_r + j\epsilon_2 E_\theta) \\ \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) &= j\omega\epsilon_0(j\epsilon_2 E_r - \epsilon_1 E_\theta) \end{aligned}\right\} \qquad (3)$$

It may be noted that Eqs. (3) are independent of the component ϵ_3.

As it stands now, the solution to (3), subject to the appropriate boundary conditions on the perfectly conducting cone and on the boundary between the plasma and the free space, presents a challenging problem. Naturally, no attempt is made to solve it as it is. Nevertheless, a significant and most practical conclusion can easily be drawn by a careful inspection of the equations in (3). If the strength of the static magnetic field B_0 is increased to a sufficiently high value such that $\omega,\omega_p \ll \omega_c$ and $\nu \ll \omega_c$, then it can be shown that $\epsilon_1 \to 1$ and $\epsilon_2 \to 0$ as $B_0 \to \infty$. In this limit, the electromagnetic fields do not experience the presence of the plasma sheath (see Fig. 1b). Moreover, the mathematical difficulties are simplified to the problem of the diffraction of electromagnetic waves by a conducting cone in free space (a topic which can be found in the existing literature).[3, 4] It is not of interest to consider the diffraction problem in this limiting situation so far as the blackout phenomenon is concerned.

Now, one may ask what happens to the plasma sheath on the surface of the cone when the static magnetic field in the azimuthal direction is increased indefinitely. The mathematical limit shows that the plasma disappears. But, in an actual situation, strong though the magnetic field may be, it is always finite. Therefore, this strong magnetic field will limit the plasma layer to a very small angle on the surface of the cone, such that the antenna is not embedded in the plasma or the attenuation caused by it will be very small. If the physical counterpart of the model described here can be achieved, then it is obvious that the blackout phenomena can be eliminated both in principle and in practice for conical structures.

Cylindrical Structure

The preceding discussion concerning a conical space vehicle shows that analytical solution of the problem is a formidable task if not an impossible one. However, whenever a space vehicle can reasonably be approximated by a perfectly conducting cylindrical structure covered symmetrically by a plasma layer, an analytical solution, although complicated, can be obtained. Since a cylindrical geometry is more convenient to study from both the practical and analytical point of view, two methods of eliminating blackout will be proposed in this configuration.

In method A, the orientation of the static magnetic field and the radiating source will be chosen in an analogous manner to its conical counterpart so that experimental work, supplemented by analysis, will establish the feasibility of applying the same principle to a conical section. In order to eliminate the blackout in this situation, the external, static-magnetic field should be in the ϕ direction of the cylindrical coordinates ρ, ϕ, z. Moreover, the electromagnetic source (i.e., antenna) should be a magnetic ring-current situated coaxially around the cylinder such that all the field components are independent of the ϕ coordinates [i.e., $(\partial/\partial\phi) = 0$]. In this case, only the components E_ρ, E_z, and H_ϕ will be excited, and they are related by the following Maxwell's equations:

$$\begin{aligned} \frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} &= -j\omega\mu_0 H_\phi - M_\phi \\ \frac{\partial}{\partial z}H_\phi &= -j\omega\epsilon_0(\epsilon_1 E_\rho + j\epsilon_3 E_z) \\ \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho H_\phi) &= j\omega\epsilon_0(\epsilon_1 E_z - j\epsilon_3 E_\rho) \end{aligned} \qquad (4)$$

where $\epsilon_{\rho\rho} = \epsilon_{zz} = \epsilon_1$, which has the same representation given by (2a), $\epsilon_{\rho z} = \epsilon_{z\rho} = \epsilon_3$, which has the same representation given by (2b), and M_ϕ is the magnetic current ring source.

Here also one may note that the previous Maxwell's equations are independent of $\epsilon_{\phi\phi} = 1 - \omega_p^2/[\omega^2(1 - j\nu/\omega)]$, which is also independent of the static magnetic field. Therefore, when $B_0 \to \infty$ such that $\nu \ll \omega_c$, $\omega_p \ll \omega_c$, and $\omega \ll \omega_c$, the influence of the plasma sheath on the propagation of electromagnetic waves disappears as previously explained in connection with the conical space vehicle. However, in the present situation, the solution of the previous set of differential equations does not involve too much mathematical complexity. In fact, a Fourier component (with respect to the kernel $e^{-j\gamma z}$) of H_ϕ can be expressed in terms of confluent hypergeometric functions or in terms of a series of the following form (the detail analysis is the subject of a subsequent paper):

$$\hat{H}_\phi = AJ_1(\eta\rho) + BN_1(\eta\rho) + \sum_{n=0}^{\infty} C_n\rho^{n+s}$$

inside the plasma sheath, and $\hat{H}_\phi = DH_1^{(2)}(\eta_0\rho)$ outside the plasma sheath, where A, B, Cn, D, η, and s are constants, and $\eta_0^2 = K_0^2 - \gamma^2$ with $K_0^2 = \omega^2\mu_0\epsilon_0$.

It seems that further work with a computer should be able to show that even when the static magnetic field is finite, the attenuation due to the high-density plasma sheath (i.e., blackout) decreases with the increase of the applied static magnetic field. This, we have already seen, will completely eliminate the blackout in the infinite limit of B_0. Furthermore, this analysis will enable one to become convinced that in the cone configuration a finite static magnetic field will decrease the attenuation of the electromagnetic wave due to a high-density plasma sheath covering the cone.

Method B closely resembles some previous analytical work performed by the author.[5] The comprehensive study of this problem will be published elsewhere. Here the radiating source is a slotted-line antenna parallel to the axis of a perfectly conducting cylinder and surrounded by a plasma layer of finite thickness. The applied static magnetic field will be parallel to the axis of the cylinder. If the length of the slot antenna is sufficiently long compared to the wavelength of the radiating frequency, the excited electromagnetic fields will consist of H_z, E_ρ, and E_ϕ only. In this situation also, the analysis shows that the blackout completely disappears in the limit of an infinite static magnetic field. Figure 1b shows the allowable frequency region for satisfactory communication during blackout in the presence of an applied static magnetic field. This information will be approximately valid for a cone configuration.

Remarks on the Implementation of a Static Magnetic Field

The foregoing analysis and discussions are based on very idealized models of the re-entry space vehicles. Without elaborating on the practical method of generating the d. c. (static) magnetic fields in the angular direction or in the axial direction of a cylindrical vehicle, the possibility of implementation of the d. c. magnetic fields was tacitly assumed. However, in practice, one does not find such ideal models. Moreover, the problem of generation of a d. c. magnetic field efficiently in an appropriate orientation may pose a stumbling block to the whole approach. Therefore, first of all we shall show that at least in principle a d. c. magnetic field in the angular direction of a cylindrical geometry is possible. Once this principle is accepted, then the second major step will be how to generate this magnetic field more efficiently in a laboratory or in an actual space vehicle.

Consider a loop of a coil carrying a d. c. current inside a cylindrical space vehicle, such that both sides of the coil are not too close to the axis of the cylinder, as shown in Fig. 1c. Because of this particular choice[6] of geometry of the coil carrying a current, the generated d. c. (static) magnetic field will have both radial and the angular components which are also functions of ρ and ϕ. The angular fields will be stronger at points such as A and B in Fig. 1c. Now, if the region A or B containing the strongest angular magnetic field is large enough in comparison with the wavelength and the size of the circumferential radiating source (slot antenna which does not cover the entire angular region 0 to 2π) which should be located on the surface of the vehicle near A or B, this situation may be considered reasonably close to the ideal case discussed in the analysis. It is anticipated that such a situation can be created in a laboratory, possibly with a little effort. One can easily see that the generation of a static magnetic field in the axial direction of a cylinder is also not very difficult, at least in a laboratory.

If in the near future, the development of very powerful electromagnets with reasonable size can be achieved, then the implementation of a static magnetic field in the angular direction of a conical or a cylindrical vehicle or in the axial direction of a cylindrical vehicle will be more practical. For practical purposes, the gap between the two pole pieces of the magnet should be large enough to include the radiating element.

Conclusion

In concluding this work, it would seem that the problem of the blackout of radio communications due to a plasma sheath can be at least eliminated in principle by a static magnetic field. Practical application of the theory requires a static magnetic-field, appropriately generated, which does not add appreciable weight to the present antenna and transmitter configuration. Further work should include such a laboratory study. It is hoped that developments of strong electromagnets using superconductors might play an important role in the problem of the radio communication blackout during re-entry. The analysis presented is appropriate for the simplified models considered here. An analysis may not be developed for an actual complicated re-entry situation. However, the simple approach made here tells us, at least, in what way the future work may be directed.

It is important to note that in all the foregoing examples, the orientation of the antenna and the static magnetic field is chosen in such a way that the field components are independent of the component of the dielectric tensor parallel to the static magnetic field. This choice enables one to control more effectively the electromagnetic waves by controlling the applied d. c. magnetic field.

References

[1] Rotman, W. and Meltz, G., (eds.), *Electromagnetic Effects of Re-Entry* (Pergamon Press, New York, 1961).

[2] Hodara, H., "The use of magnetic fields in the elimination of the re-entry radio blackout," Proc. Inst. Radio Engrs. **49**, 1827–1830 (December 1961).

[3] Siegel, K. M. and Alperin, H. A., "Scattering by a cone," Radiation Lab., Univ. of Michigan, Rept. UMM-87 (January 1952).

[4] Felsen, L. B., "Radiation from source distributions on cones and wedges," Research Rept. R-574-57, P.I.B.-502, Microwave Research Inst., Polytechnic Institute of Brooklyn, N.Y. (1957).

[5] Samaddar, S. N., "Two-dimensional diffraction in homogeneous anisotropic media," Inst. Radio Engrs. Trans. **AP-10** (September 1962); see problem #B-3.

[6] Weber, E., *Electromagnetic Fields—Theory and Application* (John Wiley & Sons, Inc., New York, 1950), Vol. 1, p. 136.

Large Displacement Analysis of Axially Compressed Circular Cylindrical Shells

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Nomenclature

A_{11}, A_{19}, A_{91}	= functions of wavelength ratio
E	= modulus of elasticity
L	= shell length
R	= mean shell radius
W	= potential energy
W_1, W_2, W_3	= extensional strain energy, bending strain energy, potential of applied axial load, respectively
a, b, e	= deflection parameters
t	= wall thickness
u, v, w	= axial, circumferential, and radial displacements, respectively, at an arbitrary point
$\bar{u}$, $\bar{v}$, $\bar{w}$	= axial, circumferential, and radial displacements, respectively, on median surface of shell
x	= axial coordinate
z	= radial coordinate
$\gamma_{x\phi}$, ϵ_x, ϵ_ϕ	= shear and normal strains at an arbitrary point
$\bar{\gamma}_{x\phi}$, $\bar{\epsilon}_x$, $\bar{\epsilon}_\phi$	= shear, axial, and circumferential strains on median surface of shell, respectively
ϵ	= unit end shortening
η	= π^2Rt/λ_ϕ^2
λ_x, λ_ϕ	= axial and circumferential half wavelengths, respectively
μ	= λ_ϕ/λ_x
ν	= Poisson's ratio (0.3)
ν_{11}, ν_{22}, ν_{12}	= functions of $\bar{u}$, $\bar{v}$, $\bar{w}$
ξ	= deflection parameter
σ	= applied average axial compressive stress
ϕ	= circumferential coordinate
χ_{11}, χ_{22}, χ_{12}	= functions of $\bar{u}$, $\bar{v}$, $\bar{w}$
ω	= $\xi\eta$

Introduction

By use of a large displacement analysis it was shown in Refs. 1–3 that buckled equilibrium configurations exist at loads considerably below the classical buckling load. In these analyses, only nonlinear terms involving the radial displacement w were included. The present note contains an investigation of the influence of the further retention of nonlinear terms involving the axial and circumferential displacements u and v on the minimum equilibrium load in the postbuckling range.

Analysis

Strain-displacement relations for the general shell derived from nonlinear theory of elasticity and Kirchhoff's assumption are given in Ref. 4 and modified in Ref. 5. These can be

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